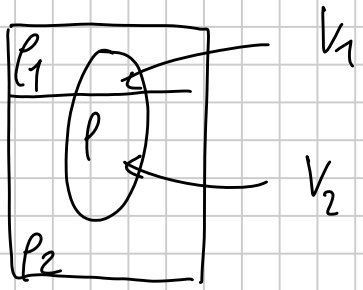
A.A. 2010/11 — 4° COMPITINO — 1/6/2011

Titolo nota 29/05/2011

1)



$\rho_1 < \rho < \rho_2$

Se è in equilibrio: $\sum \vec{F} = 0$

Scegliendo come verso positivo $\downarrow +$

$$P - S_1 - S_2 = 0$$

$$mg - \rho_1 V_1 g - \rho_2 V_2 g = 0 \qquad \text{ma} \quad m = \rho V = \rho(V_1 + V_2)$$

$$\Rightarrow \quad \rho(V_1 + V_2) = \rho_1 V_1 + \rho_2 V_2 \quad \Rightarrow (\rho - \rho_1) V_1 = (\rho_2 - \rho) V_2$$

$$\Rightarrow \quad \boxed{\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}}$$

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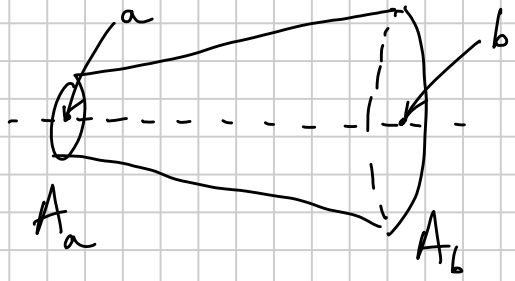
2)



$\rho = 900 \ \frac{kg}{m^3}$

$A_a = 1.9 \times 10^{-2} \ m^2$

$A_b = 9.5 \times 10^{-2} \ m^2$

$\Delta p = 7.2 \times 10^3 \ Pa \Rightarrow Q_V, Q_M = ?$

Applicando l'equazione di Bernoulli ai punti a e b

$$p_a + \rho g h_a + \frac{1}{2}\rho v_a^2 = p_b + \rho g h_b + \frac{1}{2}\rho v_b^2$$

ma $h_a = h_b \Rightarrow p_b - p_a = \frac{1}{2}\rho(v_a^2 - v_b^2)$

La portata di volume è $Q_V = Av = \underbrace{A_a v_a = A_b v_b}_{\text{eq. continuità}}$

$$\Rightarrow v_a = \frac{Q_V}{A_a}, \quad v_b = \frac{Q_V}{A_b}$$

$$\Rightarrow \Delta p = p_b - p_a = \frac{1}{2}\rho Q_V^2 \left[\frac{1}{A_a^2} - \frac{1}{A_b^2}\right] \Rightarrow Q_V = A_a A_b \sqrt{\frac{2\Delta p}{\rho(A_b^2 - A_a^2)}} =$$

$$= \boxed{77.6 \times 10^{-3} \ m^3/s}$$

$$Q_m = \rho Q_v = A_a A_b \sqrt{\frac{2\rho\Delta p}{A_b^2 - A_a^2}} = \boxed{69.8\ \frac{kg}{s}}$$

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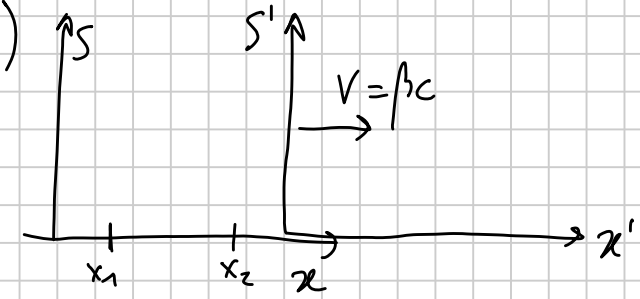
3)



A $x_1 = 1200\ m$ $t_1 = 0$

B $x_2 = 480\ m$ $t_2 = 5\ \mu s$

$x_1' = x_2'$

a) $\beta = ?$

b) $\longrightarrow \circ \longleftarrow$?

c) Per S', prima A o B?

d) $\Delta t' = ?$

a) $x_1' = \gamma(x_1 - v t_1)$ $\Rightarrow \Delta x = v\Delta t = \beta c \Delta t$

$x_2' = \gamma(x_2 - v t_2)$

$\Rightarrow \beta = \frac{\Delta x}{c\Delta t} = \boxed{-0.48}$

b) $\beta < 0 \Rightarrow$ S' si muove verso sinistra (nel verso negativo delle x)

c) $t_1' = \gamma\left(t_1 - \frac{x_1}{c^2} v\right)$ $\Rightarrow t_2' - t_1' = \Delta t' = \gamma\left(\Delta t - \frac{\Delta x v}{c^2}\right) = \boxed{4.39\ \mu s}$

$t_2' = \gamma\left(t_2 - \frac{x_2}{c^2} v\right)$

d) $\Delta t' > 0 \Leftrightarrow t_2' > t_1' \Rightarrow$ Prima A, poi B

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4) m (a riposo) $p = mc$ $\beta = ?$ $\gamma = ?$ $\frac{k}{E_0} = ?$

$p = \gamma m v = mc \Rightarrow \gamma v = c \Rightarrow \gamma\beta = 1 \Rightarrow \frac{\beta}{\sqrt{1-\beta^2}} = 1$

$\beta^2 = 1 - \beta^2 \Rightarrow \boxed{\beta = \frac{1}{\sqrt{2}} = 0.71}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \boxed{\gamma = \sqrt{2} = 1.41}$

$E = E_0 + k$ con $E = \gamma mc^2$, $E_0 = mc^2$

$\Rightarrow k = (\gamma - 1) m_0 c^2 \Rightarrow \boxed{\frac{k}{E_0} = \gamma - 1 = 0.414}$